The results of the tests with anomalously viscous liquids treated in the form of the dependence (3) are presented in Fig. 4. Here the values of $\overline{Nu}$ and ξ were reduced, in accordance with [1], to the same Reynolds numbers as for a smooth pipe.

An analysis of the dependence of Fig. 4 shows that when diaphragmmed channels are used to intensify heat exchange in anomalously viscous media their efficiency grows sharply and, under the conditions of the tests which were set up, the heat-transfer intensity exceeds the hydraulic losses by five times.

NOTATION

$\overline{Nu}$ and ξ, mean Nusselt number and coefficient of resistance in a diaphragmmed pipe; $\overline{Nu}_0$ and ξ_0, the same in a smooth pipe; Re, Reynolds number; S, spacing of diaphragm distribution; D, inner diameter of smooth pipe; d, inner diameter of a diaphragm; α, mean coefficient of heat transfer; Q, amount of heat; F, area of heat-transfer surface; $\Delta t_{\log}$, logarithmic-mean temperature head; l_i, distance between thermocouple embedding points; t_i, t_{i+1}, thermocouple readings.

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HYDRODYNAMICS AND MASS TRANSFER IN A LIQUID FILM IN THE PRESENCE OF INSOLUBLE SURFACTANTS OR INACTIVE SUBSTANCES

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Nonlinear problems on the laminar flow of a liquid film subjected to gravity and convective diffusion of substances dissolved in the fluid in the presence of insoluble active or inactive agents on its surface are examined.

The influence of substances altering the surface tension on the interface between two continuous media on the heat transfer in systems of the fluid–fluid and fluid–gas type is due to the appearance of additional tangential stresses on this boundary, which are associated with the surface tension gradient (the Marangone effect [1]). These stresses can result in an essential change in the velocity field both near the interface and in the depths of the viscous fluid, which in turn influences the convective diffusion process and the distribution of the substances mentioned on the surface itself, and therefore, on the surface-tension gradient caused by their presence. Below we call surfactants lowering the surface tension and inactive substances which raise it surfactant (SAS) in all cases when it cannot result in misunderstanding.

The influence of SAS on the mass-transfer process can be manifested because of two fundamental mechanisms. First, both the SAS itself and the homo- or heterogeneous chemical

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reactions accompanying this process cause a loss in flow stability under definite conditions, and the origination of convective cells intensifying the transfer near the surface in the long run. Similar phenomena have been investigated earlier, e.g., in [2-5]. Second, the SAS can accelerate or decelerate the motion on the surface itself, even without resulting in a loss of hydrodynamic stability, which should also be felt by the characteristics of the mass-transfer process. There is an example of an investigation of the mechanism of the second kind in [6], where the behavior of a fluid film was examined under the assumption that the limiting stage in the SAS transfer is volume diffusion. However, it was considered in [6] that the surface velocity depends quite weakly on the SAS distribution and the main physical nonlinearity, which is quite essential for the transfer process, was eliminated artificially right in the formulation of the problem. The presence of the two mentioned mechanisms of SAS action is also noted in [7].

The second mechanism is examined below under a number of simplifying assumptions in an example of an almost planar liquid film spreading laminarly under the effect of gravity, whose free surface is in contact with a gas. Plane films have been experimentally observed even in the presence of considerable surface tension gradients on the free surface, for values of the Reynolds number equal to 40-60 (see [7, 8], for example).

We consider the stationary laminar flow in a film moving along an inclined wall in the usual approximation of a thin liquid layer. The equation of motion has the simple form

$$\nu (\partial^2 u/\partial y^2) + g = 0, \tag{1}$$

where g is the effective acceleration of the external mass forces equal to the free-fall acceleration, e.g., multiplied by the cosine of the angle between the wall and the vertical. If the transverse coordinate y is measured from the wall, then the boundary conditions take the form

$$u = 0\ (y = 0), \quad \mu \frac{\partial u}{\partial y} = \delta \frac{d\Gamma}{dx}\ (y = h), \quad \delta = \frac{d\sigma}{d\Gamma}. \tag{2}$$

In writing (2) it was actually assumed that the tangential stresses in the gas can be neglected, and the quantity σ depends only on the surface concentration of the SAS. The solution of problem (1), (2) has the form

$$u = -\frac{g}{2\nu} y^2 + \left(\frac{gh}{\nu} + \frac{\delta}{\mu}\frac{d\Gamma}{dx}\right) y. \tag{3}$$

This formula, as well as the formulation of the problem itself, is valid under the conditions that the normal velocity component v is small compared to u, and u varies considerably more rapidly with y than with x. Hence, to satisfy these conditions it is necessary that

$$\left|\frac{dh}{dx}\right| \ll 1, \quad \left|\frac{d^2\Gamma}{dx^2}\right| \ll \frac{1}{h}\left|\frac{d\Gamma}{dx}\right|. \tag{4}$$

These inequalities mean that the scales of substantial changes in h and Γ along the film are much greater than h.

The surface velocity

$$U = \frac{1}{2}\frac{gh^2}{\nu} + \frac{\delta h}{\mu}\frac{d\Gamma}{dx} \tag{5}$$

can depend strongly on the SAS distribution on the surface, especially for comparatively thin films. The dependence of this velocity on the SAS surface concentration was measured in [7, 9, 10].

Considering the SAS insoluble, we write the equation of its stationary surface diffusion [11]

$$\frac{d(U\Gamma)}{dx} = D_s \frac{d^2\Gamma}{dx^2}, \tag{6}$$

where the velocity U is defined in (5).

Problems on the flow in a film and on the surface diffusion of SAS under consideration are stationary. Hence, the total volume fluid discharge and the total SAS discharge in the system are independent of x and can be considered as some given parameters. Integrating (3) over the film thickness and evaluating the first integral to (6), we then obtain the following system of two nonlinear equations:

$$\frac{1}{3}\frac{gh^3}{\nu}+\frac{1}{2}\frac{\delta h^2}{\mu}\frac{d\Gamma}{dx}=Q, \tag{7}$$

$$\left(\frac{1}{2}\frac{gh^2}{\nu}+\frac{\delta h}{\mu}\frac{d\Gamma}{dx}\right)\Gamma-D_s\frac{d\Gamma}{dx}=q,$$

which admits of the following homogeneous solution:

$$h=h_*=\left(\frac{3\nu Q}{g}\right)^{1/3},\quad \Gamma=\Gamma_*=2q\left(\frac{\nu}{9gQ^2}\right)^{1/3}. \tag{8}$$

Let us first examine the situation when the real h and Γ differ slightly from their values in (8), i.e.,

$$h=h_*+\alpha,\quad \Gamma=\Gamma_*+\beta,\quad |\alpha|\ll h_*,\quad |\beta|\ll\Gamma_*. \tag{9}$$

Linearizing (7), we have equations for α and β

$$\alpha=-\frac{1}{2}\frac{\delta}{\rho g}\frac{d\beta}{dx},\quad \left(\frac{1}{2}\frac{\delta h_*\Gamma_*}{\mu}-D_s\right)\frac{d\beta}{dx}+\frac{1}{2}\frac{gh_*^2}{\nu}\beta=0. \tag{10}$$

(For simplicity it is assumed that δ is independent of Γ, i.e., the dependence of σ on Γ is modeled by a linear function. It is clear that this assumption does not affect the matter in principle.) The general solution of (10) for β has the form

$$\beta=Ce^{-\lambda x},\quad \lambda=\frac{1}{2}\frac{gh_*^2}{\nu}\left(\frac{1}{2}\frac{\delta h_*\Gamma_*}{\mu}-D_s\right)^{-1}. \tag{11}$$

It is hence seen that the quantities h and Γ, which differ slightly in certain sections of the film from their homogeneous values in (8), will tend monotonically to these latter as x increases for sufficiently strong inactive substances so that

$$\delta>\delta^*=\frac{2\mu D_s}{h_*\Gamma_*}=(3\nu g^2 Q)^{1/3}\frac{\rho D_s}{q}. \tag{12}$$

Conversely, in the presence of active or slightly inactive substances in the system, "oscillation" of the system occurs, the homogeneous solution (8) is unstable, and its perturbations α and β grow along the film.

In the general case, by eliminating the derivative dΓ/dx from (7), we have the following relationship which relates Γ and h:

$$\Gamma=\left[\frac{qh}{2Q}-\frac{\mu D_s}{\delta h}\left(1-\frac{1}{3}\frac{gh^3}{\nu Q}\right)\right]\left(1-\frac{1}{12}\frac{gh^3}{\nu Q}\right)^{-1}. \tag{13}$$

It follows from the condition of positivity of Γ that the parameters Q and q should, in physically real situations, simultaneously satisfy the inequalities

$$Q>\frac{gh^3}{12\nu},\quad q>-\frac{2\mu D_s Q}{\delta h^2}\left(1-\frac{1}{3}\frac{gh^3}{\nu Q}\right) \tag{14}$$

or the corresponding reverse inequalities.

Differentiating (13) with respect to x and substituting the relation for dΓ/dx which follows from the first equation in (7), we obtain an explicit expression for the derivative dx/dh depending only on h. After integrating with the obvious initial conditions h = h_0 at x = 0, a relationship follows which implicitly defines the desired film thickness as a function of x. The dependence of Γ on x is found easily afterwards by using (13).

Introducing the notation

$$\xi=\frac{x}{L'},\quad \zeta=\frac{h}{L},\quad L=\left(\frac{12\nu Q}{g}\right)^{1/3},\quad L'=\frac{2\delta q}{3\rho g Q},\quad d_s=\frac{D_s}{Q}, \tag{15}$$

we write the relationship mentioned in the dimensionless form

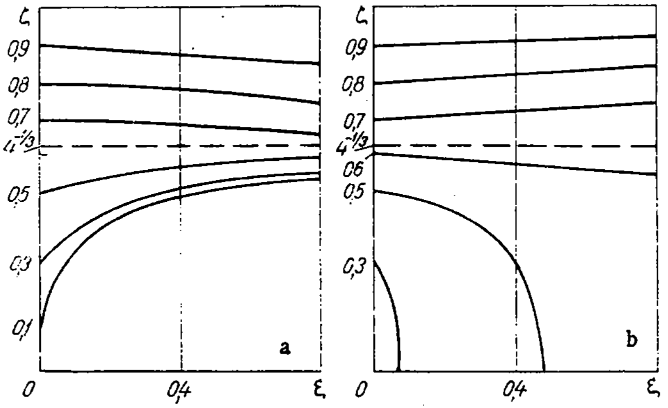

Fig. 1. Dependence of the dimensionless film thickness ζ on the dimensionless longitudinal coordinate ξ for inactive (a) and active (b) substances while neglecting their surface diffusion; the dashed curve is $\zeta = \zeta_* = 4^{-1/3} \approx 0.63$.

$$\frac{L'}{L}\,\xi = d_s\left[\frac{5}{2}\,\zeta - \frac{13}{18}\,\frac{\zeta}{1-\zeta^3} + \frac{17}{4^{1/3}\,18}\,\ln\frac{(1-4^{1/3}\zeta)^2}{1+4^{1/3}\zeta+4^{2/3}\zeta^2} - \right.$$
$$- \frac{55}{108}\,\ln\frac{(1-\zeta)^2}{1+\zeta+\zeta^2} - \frac{17}{3^{3/2}\,4^{1/3}}\,\operatorname{arctg}\frac{4^{1/3}\,2\zeta+1}{3^{1/2}} +$$
$$\left. + \frac{55}{3^{1/2}\,18}\,\operatorname{arctg}\frac{2\zeta+1}{3^{1/2}}\right] + \frac{L'}{L}\left[\ln\frac{1-\zeta^3}{1-4\zeta^3} - \frac{1}{2(1-\zeta^3)}\right]\operatorname{sign}\delta + C, \tag{16}$$

where the constant of integration is expressed easily in terms of $\zeta_0 = h_0/L$ from the initial condition.

An investigation of (16) and the relationships for Γ, U, etc., is quite awkward; it is apparently expedient to execute it in application to simplified particular cases and situations which are of direct practical interest. Thus, the SAS diffusion can sometimes be neglected; in this case we obtain from (16)

$$\xi \approx \left[\ln\frac{(1-\zeta^3)(1-4\zeta_0^3)}{(1-\zeta_0^3)(1-4\zeta^3)} - \frac{1}{2}\left(\frac{1}{1-\zeta^3} - \frac{1}{1-\zeta_0^3}\right)\right]\operatorname{sign}\delta, \tag{17}$$

and from (5) and (13)

$$\gamma = \frac{\Gamma}{\Gamma_*} \approx \frac{3}{2^{4/3}}\,\frac{\zeta}{1-\zeta^3}\,, \quad V = \frac{Uh_0}{2Q} \approx \frac{\zeta_0(1-\zeta^3)}{\zeta}\,. \tag{18}$$

The quantities L and L' can be considered as linear scales of variation of h and the characteristic distance (on which h varies substantially), respectively.

The dependences of ζ on ξ are shown in Fig. 1 for different $\zeta_0 < 1$ [ζ cannot be greater than one in conformity with inequalities (14)]. For inactive substances [in conformity with the above relative to (12)], a buildup of the homogeneous state (8) is observed; for $x \to \infty$ ($\xi \to \infty$) we have $h \to h_*$ ($\zeta \to \zeta_* = 4^{-1/3}$), $\gamma \to 1$, $V \to V_* = 4^{-2/3}3\zeta_0$. For active substances, the stationary regime of the type considered above is possible only for films of bounded extent. Thus, if $\zeta_0 < \xi_*$, then the length of such a film cannot be greater than

$$x_m = L'\xi_m, \quad \xi_m = -\frac{1}{2}\left(\frac{1}{1-\zeta_0^3} - 1\right) + \ln\frac{1-\zeta_0^3}{1-4\zeta_0^3}\,. \tag{19}$$

For $\zeta_0 > \zeta_*$, the quantity ξ_m is a still more complex function of ζ_0. If the length of the film containing the active substance is greater than x_m, then stationary flows and diffusion are generally impossible. In this case, the processes mentioned apparently go into the wave regime, solitary waves or groups of waves being propagated downstream appear. The generation of such waves should start for values of x less than x_m, which on the whole indicates instability of the stationary regime considered above for $\delta < 0$. An investigation of the instability of the stationary and the origination of the wave regimes is a separate problem and outside the scope of this paper.

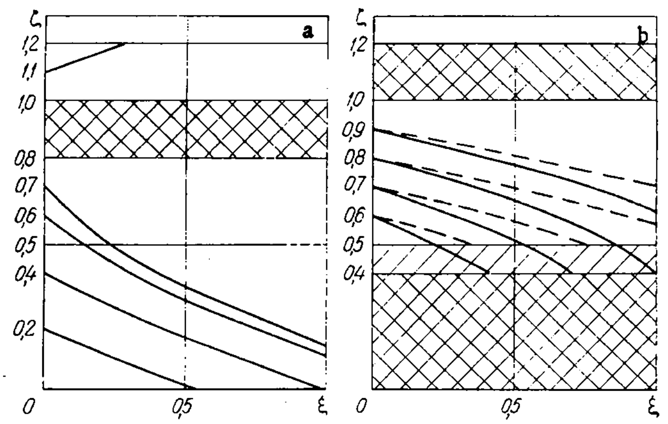

Fig. 2. Dependence of ζ on ξ for a thin film in the presence of inactive substances for δ/δ* = 0.433, ζ° = 0.8 (a) and active substances (b) for δ/δ* = −1.23, ζ° = 0.4, and δ/δ* = −0.53 (solid curves and dashed curves, respectively). The "forbidden" regions of values of ζ are hatched.

In order to illustrate the influence of the surface diffusion of SAS, let us consider the limiting case of a very thin film when the following inequalities are satisfied:

$$\left|\frac{d\Gamma}{dx}\right| \gg \left|\frac{\rho g h}{\delta}\right|, \quad \left|\frac{d\Gamma}{dx}\right| \gg \frac{g h^2}{\nu D_s}\,\Gamma. \tag{20}$$

(The first of these inequalities means that surface motion stimulated by the surface tension gradient plays the fundamental part in producing the fluid stream in the film.)

Let us emphasize that the limiting case under consideration is also important for small g, i.e., for almost horizontal films under reduced gravitation conditions. Then inequalities (14) become

$$1 > \zeta, \quad 1 > -\frac{1}{2^{1/3}3}\,\frac{\delta^*}{\delta}\,\frac{1-4\zeta^3}{\zeta^2} \quad (\delta^* > 0), \tag{21}$$

where δ* is defined from (12) and instead of (17) we obtain

$$\xi \approx \frac{3}{2}\left[\zeta^3 - \zeta_0^3 - \frac{1}{2^{1/3}}\,\frac{\delta^*}{\delta}\,(\zeta - \zeta_0)\right] \operatorname{sign}\delta, \tag{22}$$

and instead of (18)

$$\gamma \approx \frac{1}{1-\zeta^3}\left(\frac{3}{2^{4/3}}\,\zeta + \frac{1}{2^{5/3}}\,\frac{\delta^*}{\delta}\,\frac{1-4\zeta^3}{\zeta}\right), \quad V \approx \frac{\zeta_0}{\zeta}. \tag{23}$$

[Let us recall that a situation when both inequalities in (21) are replaced by the reverse is also possible.]

Let us examine the expression

$$\zeta^3 - \frac{2^{1/3}3}{4}\,\frac{\delta}{\delta^*}\,\zeta^2 - \frac{1}{4} = 0, \quad \Delta = \frac{1}{64}\left[1 + \frac{1}{2}\left(\frac{\delta}{\delta^*}\right)^3\right], \tag{24}$$

where Δ is the determinant of this equation. Examining the sign of the determinant, we see that for $\delta/\delta^* > -2^{1/3}$, $\delta/\delta^* = -2^{1/3}$, $\delta/\delta^* < -2^{1/3}$ Eq. (24) has one, two, and three real roots, respectively. In conformity with inequalities (21) and their reverse inequalities, these roots determine the "forbidden" ranges of variation of ζ. Let ζ° be a root of (24). Then for δ > 0 this root is always unique and only $\zeta_* < \zeta°$ is possible, where it follows from (21) and the reverse inequalities that ζ cannot be in the range [ζ°, 1]. Analogously, for δ < 0, the root of (24) is always less than ζ_*; in this case ζ cannot lie in the range [0, ζ°] as well as in the domain $\zeta \geqslant 1$.

The characteristic dependences of ζ on ξ, corresponding to (22), are illustrated by the curves in Fig. 2. This formula does not describe the emergence into the homogeneous state (8) even when inequality (12) is satisfied, which is evidently associated with the spoilage of (20) for the validity of (22) upon approaching such a state. Therefore, the curves in Fig. 2 terminate at finite ξ_m. For $\delta > \delta^*$ this indicates that (22) ceases to be true when ξ approaches ξ_m, while for $\delta < \delta^*$ it reflects the absence of a stationary regime for extended films (see the analysis of the curves in Fig. 1). These curves are related principally to the beginning of the flow domain corresponding to small ξ.

The dependences of ξ on the dimensionless SAS concentrations and the velocity on the film surface are also clear from the curves in Figs. 1 and 2 and (18) and (23), respectively.

Let us note that all the results have been obtained under the assumption that inequalities (4) are valid. It is easy to write these inequalities in terms of the scales L, L' and the quantities ξ, ζ introduced earlier. That they are satisfied should be confirmed in each specific case.

Now, let us investigate the influence of the SAS and the corresponding changes in velocity in the film on the mass transfer of the film to the surrounding gas. For simplicity, we assume that the diffusing substance is neutral (i.e., does not influence the surface tension and does not go into chemical reaction), the rate of mass transfer is limited by the kinetics of diffusion within the film, and the appropriate Peclet number is large compared to one. Then the diffusion boundary layer at the film surface can be considered thin and u(x, y) can be replaced by U(x) within its limits, $v(x, y) \approx y' dU/dx$ can be obtained from the fluid continuity equation by taking account of the inequalities in (4), where $y' = h - y$, and the conditions of agreement with the constant concentration c_0 for $y' = 0$ and disappearance as $y' \to \infty$ can be taken as the boundary conditions for the concentration c of the diffusing substance. We consequently arrive at the following convective diffusion problem:

$$U(x) \frac{\partial c}{\partial x} - y' \frac{dU(x)}{dx} \frac{\partial c}{\partial y'} = D \frac{\partial^2 c}{\partial y'^2}, \qquad (25)$$
$$c = c_0, \quad y' = 0; \quad c \to 0, \quad y' \to \infty.$$

Introducing the thickness of the diffusion boundary layer dependent on the longitudinal coordinate x in the usual manner and the self-similar variable which is the ratio of y' to this thickness, we obtain the solution to the problem (25) after simple calculations in the form [12]

$$c = c_0 \operatorname{erfc}\left[\frac{y'}{2}\left(\frac{U}{D}\right)^{1/2}\left(\frac{1}{U}\int_0^x U(t)\,dt\right)^{-1/2}\right]. \qquad (26)$$

In particular, if U does not vary very strongly with x, we have a simplified formula from (26)

$$c \approx c_0 \operatorname{erfc}\left[\frac{y'}{2}\left(\frac{U}{Dx}\right)^{1/2}\right]. \qquad (27)$$

Expressions for the diffusion flux of a substance on a film corresponding to (26) and (27) have the form

$$J = c_0\left(\frac{DU}{\pi}\right)^{1/2}\left(\frac{1}{U}\int_0^x U(t)\,dt\right)^{-1/2} \approx c_0\left(\frac{DU}{\pi x}\right)^{1/2}, \qquad (28)$$

from which it is seen that the SAS affecting the surface velocity thereby influences the intensity of the film mass transfer to the surrounding medium. The dependence of J on the different parameters can therefore be obtained on the basis of the results presented above. In particular, (18) and (23) can be used for U for the cases considered of a weak surface SAS diffusion and a thin film.

It is convenient to introduce the diffusion flux density J_* of the substance on a film of constant thickness h_* corresponding to a given value of Q in conformity with (8), whose SAS concentration on the surface is independent of x. It is clear that the quantity J_*, which can be determined from (28), where U is evaluated by means of (18) with $\zeta = \zeta_*$, is independent of Γ_* and hence, in particular, describes the diffusion flux on a film which does not generally contain SAS. Neglecting the surface diffusion, we have from (18) and (28)

$$j = \frac{J(\zeta)}{J_*} \approx \frac{2^{2/3}}{3^{1/2}}\left(\frac{1-\zeta^3}{\zeta}\right)^{1/2} \approx 0.92\left(\frac{1-\zeta^3}{\zeta}\right)^{1/2}. \qquad (29)$$

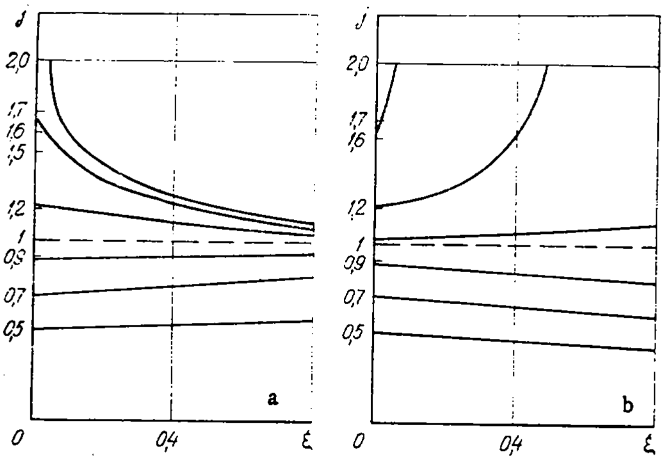

Fig. 3. Dependence of the dimensionless flux density j on ξ with surface diffusion, corresponding to the curves in Fig. 1, neglected; the dashed curve is j = 1.

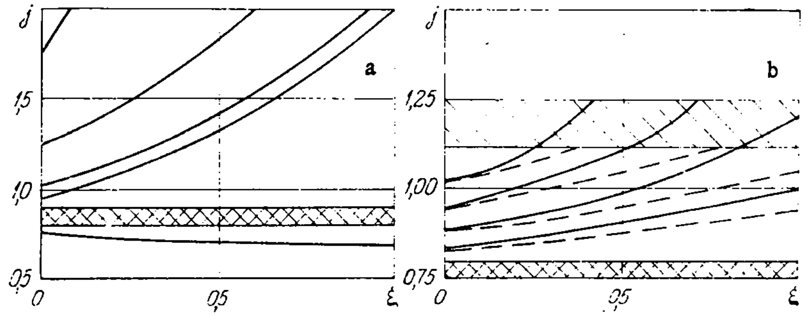

Fig. 4. Dependence of j on ξ for a very thin film corresponding to the curves of Fig. 2: a) hatched region within the limits j = 0.79-0.89; b) j = 0.75-0.79 and j = 1.12-1.25.

The dependences of j on ξ, computed for the quantities $\zeta = \zeta(\xi)$ shown in Fig. 1, are presented in Fig. 3. As is seen, the active and inactive substances intensify the mass transfer if the initial dimensionless film thickness is $\zeta_0 < \zeta_* = 4^{-1/3}$ and reduce it in the opposite case.

In the thin film approximation, when (20) are valid, we analogously obtained from (23) and (28)

$$j = \frac{J(\zeta)}{J_*} \approx \frac{2^{2/3}}{3^{1/2}\zeta^{1/2}} \approx \frac{0.92}{\zeta^{1/2}} \,. \qquad (30)$$

The dependences of j on ξ corresponding to the curves in Fig. 2 are shown in Fig. 4. Here both the active and the inactive substances are capable of altering the mass transfer characteristics significantly.

The characteristic distances along the films on which the influence of the initial conditions is felt on its flow mode, or the buildup occurs of a nonstationary wave mode for films contaminated by surface inactive and active substances, are of the order of L'. Hence, the influence of the inactive substances on the intensity of mass transfer is substantial if the extent of the film section with the active mass transfer is less than or commensurate with L', for which it is necessary that the quantity δ be sufficiently large. On the other hand, the influence of the active substances can be substantial, in principle, for both large and small $|\delta|$; in this connection it is of indubitable interest to investigate the possible wave flow modes due to the presence of such substances on the film surface and being realized on relative extended films.

In conclusion, let us emphasize that, as follows from the results presented, the contamination of a film surface in real film mass transfer apparatus by surfactant of inactive substances is not always an undesirable phenomenon because it even turns out to be quite useful for the intensification of the mass transfer being conducted in the apparatus in a number of cases.

NOTATION

c, concentration of the diffusing impurity; D, its coefficient of diffusion; D_s, d_s, dimensional and dimensionless surface diffusion coefficients; g, acceleration of the external mass forces; J, j, dimensional and dimensionless diffusion flux density; h, film, thickness; L, L', linear scales; Q, q, fluid and surfactant substance discharge; u, velocity; U, V, dimensional and dimensionless surface velocities; x, y, y', longitudinal and transverse coordinates; α, β, perturbations of the homogeneous flow mode; Γ, γ, dimensional and dimensionless surfactant substance concentration; Δ, determinant in (24); δ, δ^*, quantities defined in (2) and (12); ζ, dimensionless film thickness; λ, quantity introduced in (11); μ, ν, dynamic and kinematic fluid viscosities; ξ, dimensionless longitudinal coordinate; ρ, fluid density; σ, coefficient of surface tension; the subscript zero refers to quantities in the initial section of the film, the asterisk subscript denotes quantities referring to the homogeneous flow mode.

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RHEOLOGY OF STRETCHING FOR POLYMER LIQUIDS

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UDC 532.5:532.135

Theoretical and experimental studies are presented on various effects in uniform isothermal stretching of elastic liquids.

This theoretical discussion of stretching is based on the rheological equations for an incompressible elastic fluid [1]; these equations give a close quantitative description of the behavior of such liquids under shear [2]. The minimum necessary number of rheological constants is four, and then the equations take the form:

$$\sigma' + p\delta = \eta s \exp\left(-\frac{\beta}{\mu} W_s\right) \mathrm{e} + 2\mathrm{c}W_1 - 2\mathrm{c}^{-1} W_2; \tag{1}$$

$$\mathrm{c}^{\nabla} - \mathrm{ce} - \mathrm{ec} + 2\mathrm{ce}_p^* = 0; \quad \mathrm{c}^{\nabla} = \dot{\mathrm{c}} + \omega\mathrm{c} - \mathrm{c}\omega; \tag{2}$$

$$\mathrm{e}_p^* = \frac{1}{2\mu\theta} \exp\left(-\frac{\beta}{\mu} W_s\right)\left[\left(\mathrm{c} - \frac{I_1}{3}\delta\right) W_{s1} - \left(\mathrm{c}^{-1} - \frac{I_2}{3}\delta\right) W_{s2}\right];$$

$$I_1 = \mathrm{Sp}\,\mathrm{c}; \quad I_2 = \mathrm{Sp}\,\mathrm{c}^{-1}; \quad 2\mu = \frac{\eta}{\theta}(1 - s); \tag{3}$$

$$2W_s = W(I_1, I_2) + W(I_2, I_1); \tag{4}$$

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